

Math Progress Resources

Assess Clearly 

+ Respond Progressively

= Multiply Progress

Developed for Matteson School District 162

Dr. Blondean Davis, Superintendent

by the DePaul University Center for Urban Education

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Action Plan

Week	Action
<i>February 1</i>	<p>Introduce (show me) a math problem solving strategy scaffold to demonstrate this week's math focus.</p> <p>(Help me) Students complete the strategy scaffold collaboratively/with guidance with another problem.</p> <p>Teacher responds with "push-ups". Plus _____</p>
<i>February 8</i>	<p>Students complete math problem strategy scaffold, then pair, compare, "repair".</p> <p>Watch me, then Help me: Teacher responds with "push-ups".</p> <p>Plus _____</p>
<i>February 15</i>	<p>Students complete math problem strategy scaffold, as independent assessment.</p> <p><i>Help me</i> move ahead: Teacher responds with "pull-ups" and "push-ups".*</p> <p>Plus _____ **</p>

**Bring completed examples to the workshop on February 20th.*

****PLUS:** Bring examples of YOUR great TEACHING strategies to the February 20th session.

MATH MIX: New and Continuing PRIORITIES

Research confirms that if the math curriculum includes “frequent cumulative review” that enables students to retain greater math competence. Among sources supporting this “mix” is the report “Assisting Students Struggling with Mathematics” of the What Works Clearinghouse, IES Practice Guide, US Department of Education. This chart is designed to organize planning for new math content and inclusion of math learned earlier in the school year in activities such as: learning centers; “bell ringers”; homework, art, science, social science--*Integrating math into science and social science makes math more meaningful.*

Math Practice Standards should be emphasized

—particularly standard 1: **Make sense of problems and persevere in solving them.**

Week of	New Math	Math “Mix”—Content to Revisit

Homework Essential: Students need to take home an example of how to solve problems—that teachers prepare or that *they prepare* so they can practice correctly.

Daily kinds of assessment:

__glossary __journal __my own example __change the problem, solve it

Weekly kinds of assessment:

__solve problem, explain patterns and strategies __write math booklet

__make my own “anchor chart” __make “math path”—steps to solution

Table 2. Recommendations and corresponding levels of evidence

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Moderate
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	Low
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

Source: Institute for Education Science, US Department of Education
 “Assisting Students Struggling with Mathematics”

Principles of Assessment for Effective Teaching/Learning

- ☐ Focus: Emphasize important elements (essential and enduring content/core abilities)
- ☐ Application: If the assessment focuses on new knowledge, the application should use already developed skills. If the assessment focused on new abilities, it should require the student to apply them in new but familiar kinds of situations.
- ☐ Explicit: Clearly communicate the intended outcomes, the kind of work to be done, and the criteria or standards to be met
- ☐ Flexibility: Use a variety of formats to assess
- ☐ Developmental for the learner: Provide opportunities for learners to improve based on clear and explicit response to their work products
- ☐ Formative for the educator: provide a basis to analyze and improve instruction.
- ☐ Transfer: Evaluate learner's ability to apply what has been learned to other contexts
- ☐ Independence: Evaluate the degree of independence with which a learner uses the knowledge and skills

Solve Learning Problems

Usually, the obstacle is not one isolated skill—how do you solve these learning potholes?

Problem	Solutions
Student has difficulty staying on task.	<ol style="list-style-type: none"> 1. Ask student to restate directions 2. Write directions on board. 3. Students work in pairs.
Student cannot work independently.	
Student is not interested.	

What problem will the homework assignment on the next page solve?

My Own Math Homework Guide

Complete this guide at school.

Use it to help you solve math problems at home.

Kind of Problem: _____

Steps to Solve It:

Put an example of a good solution on the back of this page.

I applied this math practice standard to accomplish this progress:

Keep it simple, Smarty.



Check each strategy that would help all students learn.

- ☐ Chunk the content or skill into segments
- ☐ Teach with clear focus on one skill, strategy, or topic at a time.
- ☐ Model the strategy—think out loud.
- ☐ Scaffold Learning with:
 - ☐ Clear directions that you explain and post
 - ☐ Step-by-step activities
 - ☐ Student learning “organizers”—activity guides for students to complete
 - ☐ Student learning “partners”
- ☐ Diversify assessment with a variety of ways for students to demonstrate learning
- ☐ Frequently check and respond to student learning
- ☐ Explain directions and give concrete examples
- ☐ Maintain frequent eye contact
- ☐ Give verbal directions in clearly stated steps
- ☐ Test one concept at a time
- ☐ Walk by student’s desk to check for accuracy and on task behaviors
- ☐ Write assignments and give verbal instructions
- ☐ Provide visual aids
- ☐ Give simple directions with written examples
- ☐ Ask student to explain what you said in his/her own words
- ☐ **Reinforce previously mastered skills**

Teach Strategically

Which of these Powerful Practices PREVENT learning problems?

Differentiation Strategies--The following list was compiled based on IES What Works studies and is included in Powerful Practices for High Performing Special Educators (Roberta C. Kaufman and Robert W. Wandberg, editors, Corwin Press, 2010).

- ☐ Cooperative Learning Students work as a team to accomplish a task
- ☐ Curriculum-Based Probes Student performance of skills that are timed and then charted to reflect growth
- ☐ Direct Teaching of Vocabulary--Specific vocabulary instruction using a variety of activities that hold attention
- ☐ Explicit Timing--Timing of seatwork to increase proficiency
- ☐ Graphic Organizers -- Visual display of information to structure concepts and ideas
- ☐ Peer Tutoring--Pairing students, with one trained to tutor the other
- ☐ Preassessment Organization Strategies --Use of specific practices designed to reinforce student's recall of content
- ☐ Reciprocal Peer Tutoring --Pairing students who then select a team goal and tutor each other
- ☐ Specific Informal Assessments --Use of a variety of methods including questioning for retention
- ☐ Teacher Think-Alouds--Explicit steps are modeled out loud in order to develop steps in problem solving processes
- ☐ Using Short Segments to Teach Vocabulary--Short time segments are used to teach vocabulary through listening, speaking, reading, and writing
- ☐ Using Response Cards During Instruction--Students write brief answers to teacher questions and hold them up so teacher can review answers

What learning problems could the following three activities reduce?

Math Interest Survey

1. What is an important math skill?
2. What is a good way to learn math?
3. What kind of math is difficult?
4. What kind of math is easy?
5. What do you like about math?
6. What don't you like about math?

Put your answers in the boxes. Your teacher can organize this as a project—your class can make a bar graph if you cut the boxes and sort them with other students.

1	2
3	4
5	6

This Week's Math

This graphic organizer applies to all math standards and is designed to guide students' clarifying of what they learn in math each week.

Topic: _____
(Write what the focus of the work this week was.)

What are some important words to know when thinking about this math topic?
There are three columns. If the word also can be shown as a symbol, put that symbol in the third column.

Word	What It Means	Symbol

What's important to know about this math topic?

In Your Own Words Name: _____

Solve this problem: Math problem solvers need to be able to use the language of math. Write an explanation then give an example of what each math word means.

WORD: _____

Explanation	Example

WORD: _____

Explanation	Example

WORD: _____

Explanation	Example

Effective Strategies: What Works?

The top five highly rated strategies in these content disciplines are as follows:

Reading:

1. Pre-assessment organization strategies
2. Graphic Organizers
3. Cooperative Learning
4. Direct Teaching of Vocabulary
5. Specific Informal Assessments

Math:

1. Curriculum-Based Probes
2. Reciprocal Peer Tutoring
3. Graphic Organizers
4. Explicit Timing
5. Teacher Think-Alouds

Science:

1. Curriculum-Based Probes
2. Graphic Organizers
3. Peer Tutoring
4. Using Short Segments to Teach Vocabulary
5. Using Response Cards During Instruction

Powerful Practices for High Performing Special Educators

The editors explain that the following strategies were determined to be effective in these core disciplines. They note that...the following common principles are also associated with the practices:

- The practices promote efficient use of time with routines and expectations identified.
- The practices utilize teacher modeling.
- The practices encourage student engagement in the learning process.
- There is documentation of effectiveness.

Math Path

Common Core Math Practice Standard 1: Make sense of problems and solve them persistently.

What is the problem?

Steps to solve it.

Solve the problem here.



Common Core Math Practice Standard 2: Reason abstractly and quantitatively.

This is an important math idea that this solution shows.

Chart to Make Math Patterns Clear

CCSS Math Practice Standard 8. Look for and express regularity in repeated reasoning.

number	operation	number	=	result
6	x	2	=	12
6	/	2	=	3
6	x	3	=	18
6	/	3	=	2
			=	
			=	

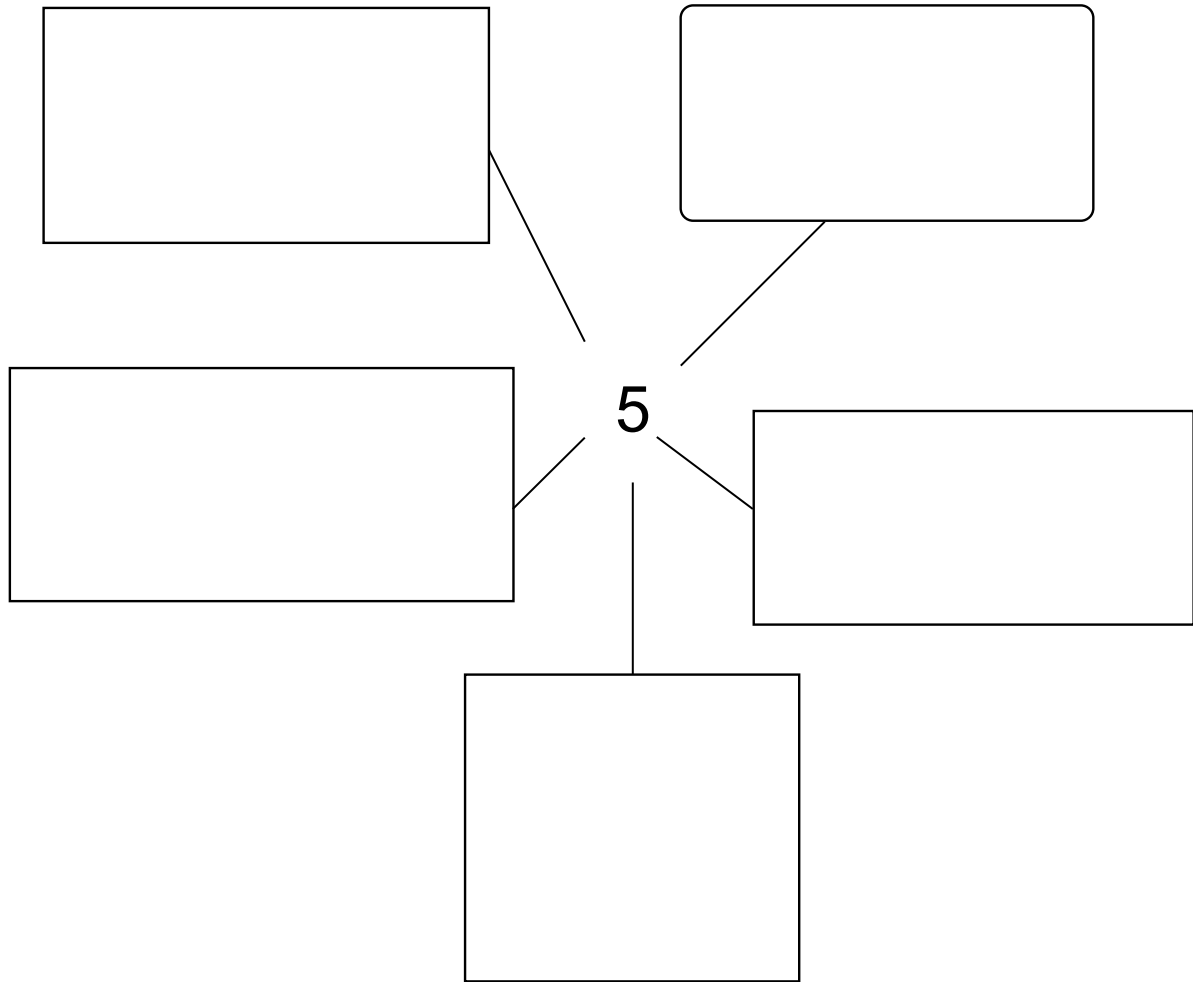
Chart Algebra

number	operation	number	=	result
a		b	=	c
5			=	8
25			=	100

Five Ways to Make a Five

CCSS Math Practice Standard 2. Reason abstractly and quantitatively.

Task: Use the math you know to show five different ways to make equations that result in 5.

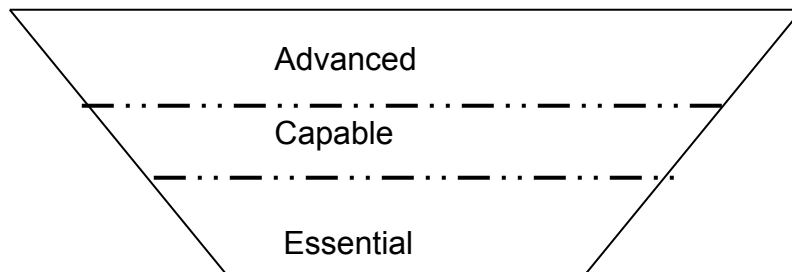


Move from Simpler to Complex

First you choose a chunk—important content or skill.

The Chunk—what will you teach?

Then you set up activities at increasing levels of complexity.



Advanced

Students' demonstrate "fluency" with the skill or content. Their work is a "model" for the application of the skill or clarification of the content. Usually involves not only critical thinking but also synthesis.

Capable

Students can apply skill or communicate knowledge accurately and in their own words independently and in a different context from the situation in which it was learned.

Essential

Students know the content you have taught and can state it accurately (usually re-state as learned) or apply it correctly in basic context (same as the content it was learned).

Is this a grading scale or a learning progression?

For additional information and resources on the Layered Curriculum:

Brains.org--<http://www.brains.org>

Layered Curriculum, The practical solution for teachers with more than one student in their classroom by Kathie F. Nunley

Teaching With The Brain In Mind by Eric Jensen

*This example includes **activities** at each level.*

The Chunk: How to ... (add fractions; graph an equation...

The levels of the Layered Curriculum can correlate with grades.

A--Exemplary

Write a guide to this part of math, including two examples of problems and their solutions.

B--Got it!

Solve a problem correctly and explain how you solved it and why you used this strategy.

C--Can Do: Capable

Solve a problem correctly and show your work.

Move to Essential:

D--Developing: Need support to follow steps.

There is no F for Fail. F is for FIX UP.

**Each of the A, B, C activities results in a product.
That product can be an activity or an assessment.**

What's the difference?

If it's an **assessment**, students complete it **independently**.

If it's an **activity**, students *collaborate* or receive *guidance*.

Chunk: _____

Exemplary!

Got it!

Can do!

Developing...

You can use the layered curriculum as a developmental progression.

This week's focus: _____

Monday	Tuesday	Wednesday	Thursday	Friday
Make it Clear	Take it Farther	Work with It	Think it Through	Fix and Finish UP
Act	Act	Act	Act	Act UP
Assess: Check for Clarity	Assess: Check for Understanding	Assess: Check for Understanding	Check for Independent Competence	Confirm Independence

Teach Strategically. Assess to Advance!

Focus Clearly

- ☐ Survey students—pre-assess for learning interests as well as knowledge, skills.
- ☐ Set explicit objectives and criteria.
- ☐ “Show me”—demonstrate step-to-step—with graphic organizer or other visuals.

Develop with Depth

Teacher:

“Help me”

Guide students in small groups, pairs, individually.

- ☐ Ask questions that require students to tell “how”—strategies.
- ☐ Give students “think time” (not wait time).
- ☐ Use “response cards” that students raise to respond.
- ☐ “Remind me”—post directions with example.
- ☐ Differentiate complexity of the task with increasing challenge.

Students... “Watch us” (you do, collaboratively)

- ☐ Collaborate to solve problems.
- ☐ Respond to a question individually, then pair to compare.
- ☐ After they pair to compare, they repair (improve response).
- ☐ Make their own “how to” examples.

Assess, then Respond to Advance

“Watch me”

- ☐ Students set a goal for the week; end of week students report their progress.
- ☐ Teacher checks for understanding and misunderstanding during class as well as at end of class.
- ☐ Differentiate kinds of assessments.
- ❖ Student-made math glossary.
- ❖ Use graphic organizers to assess formatively.
- ❖ Use graphic organizers to assess summatively.
- ❖ Students make weekly report—how do you...?

Teacher gives specific feedback—how to improve (Pull up) and advance (Push up).

“Let me”

- ☐ Students complete booklets, reports, exhibits.
- ☐ “Let me”...
- ☐ Students take math “walk”, view what other students have done, post their notes of what’s working and ways to go farther.

Assess to Advance

Which assessments do you currently use?

Each of these would result in a product.

- ☐ Make math glossary.
- ☐ Daily “learning report” (also called “exit ticket”) *in response to specific question.*
- ☐ Choose the best answer (multiple choice), then justify that choice.
- ☐ Draw the problem and solution.
- ☐ Explain math rule or pattern with example.
- ☐ Change math problem—adjust the specifics and then give to students to solve after you demonstrate “how to”.
- ☐ Write a math guide with example.
- ☐ Make a math “book” or display.
- ☐ Complete graphic organizers: __list, __strategy sequence,
__ “math path”, __diagram, __flow chart, _____
- ☐ _____
- ☐ _____

Principles of Assessment for Effective Teaching/Learning

- ☐ Focus: Emphasize important elements (essential and enduring content/core abilities)
- ☐ Application: If the assessment focuses on new knowledge, the application should use already developed skills. If the assessment focused on new abilities, it should require the student to apply them in new but familiar kinds of situations.
- ☐ Explicit: Clearly communicate the intended outcomes, the kind of work to be done, and the criteria or standards to be met
- ☐ Flexibility: Use a variety of formats to assess
- ☐ Developmental for the learner: Provide opportunities for learners to improve based on clear and explicit response to their work products
- ☐ Formative for the educator: provide a basis to analyze and improve instruction.
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Teachers are problem solvers.

How do you solve learning problems?

First you locate them!

Assessments locate problems.

I do, we do, you do...YOU DO BETTER!

PULL UPS	PUSH UPS
<ul style="list-style-type: none"> ❑ Peer coach. ❑ Teacher models, step to step. ❑ Students model. ❑ Give clear written steps to follow. ❑ Give examples—more than 1. ❑ Students work in pairs—compare their work, then “repair”—improve it. ❑ “Break down” the content or skill—break it into smaller parts using task analysis. Then guide through each step. ❑ Partially complete a graphic organizer. 	<ul style="list-style-type: none"> ➤ Write a math guide. ➤ Make up math problems—and provide solutions. ➤ Make a math display. ➤ Make a presentation ➤ Make a math model. ➤ Make a math booklet. ➤ Use math to design with precision. ➤ _____ ➤ _____

Assessment Templates

The following templates can assess specific content, skills and strategies and Math Practice standards 1 and 6.

Common Core STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.

- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.

6. Attend to precision.

- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

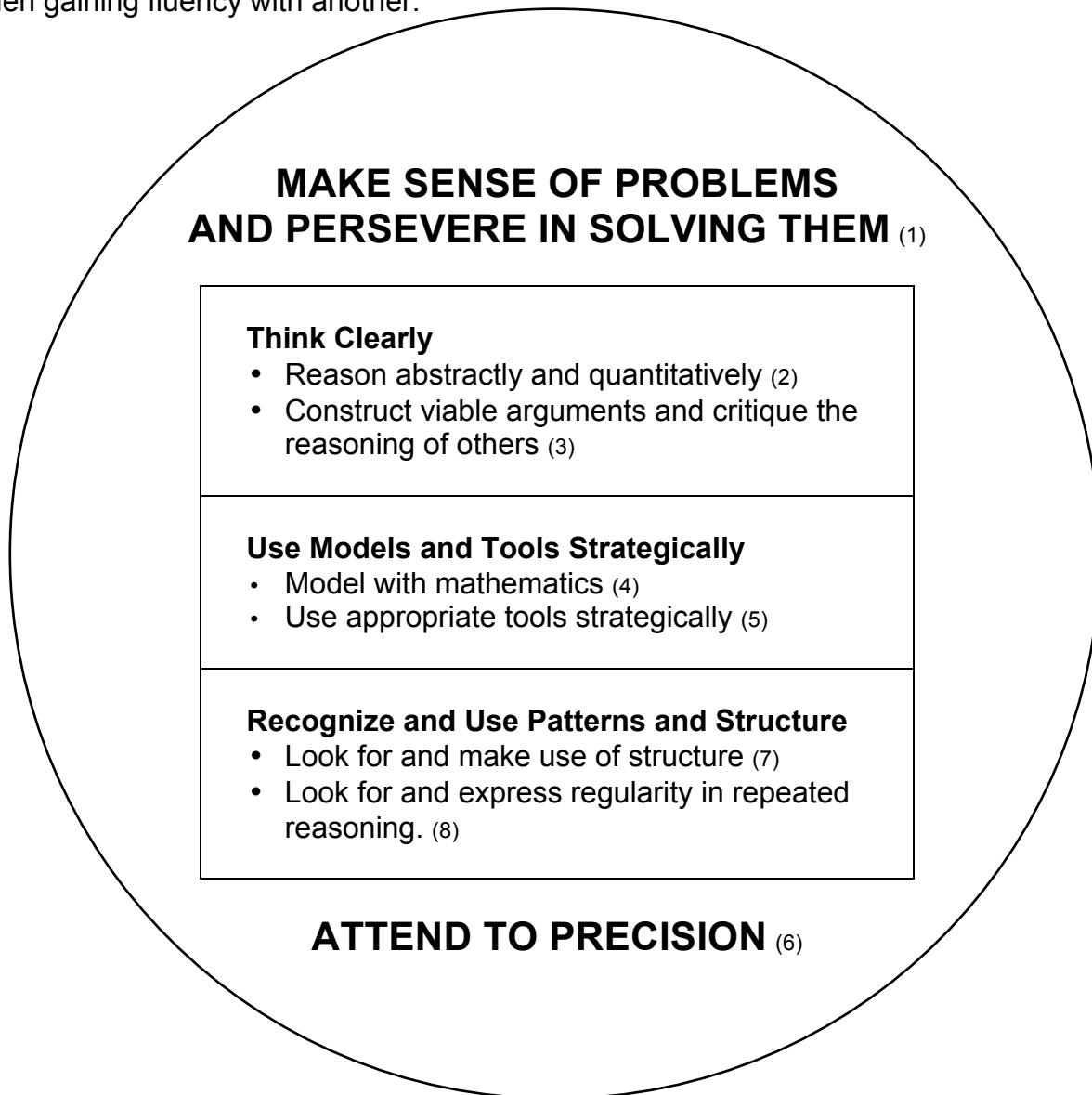
What do those practice standards mean?

That finding an answer is not the destination—it's how you get there and the patterns you find as you solve a problem.

Strategic Problem Solvers Apply the Common Core Math Practice Standards

It's about thinking clearly.

Two of the standards are essential every time students solve any problem, so they are outside the boxes. The standards in the boxes are important, but students need to move into the standards progressively, making one standard a continuing habit and then gaining fluency with another.



This diagram by the Center for Urban Education is based on "Grouping the SMPs" (McCallum 2011), Supporting Student Success, the Indiana Department of Education.

Build Graphic Organizers into the Gradual Release of Responsibility

Show me! (Teacher models.)

Use a graphic organizer to model—explain how it organizes thinking, how students put information from text or math source into a structure that shows relationships.

Help me! (Teacher guides.)

Complete organizer with new text or math problem **with** group or individual.

We help each other! (Students collaborate.)

Collaboratively complete organizer.

Watch me! (Student works independently.)

Independently student uses the organizer with new content/problem.

Let me! I move ahead!

Design my own ...step to step guide, booklet, question (with explanation of one way to solve it—plus the answer)

Structure Strategic Problem Solving

Five classes are going on a bus trip, and each class has 21 students. If each bus holds only 40 students, how would you figure out how many buses are needed for the trip?

1. What are you going to figure out?	
2. How will you solve the problem?	
3. What information will you use?	
4. What do you estimate the answer will be?	

Check your estimate by solving the problem here.

Math Assessor

Student: _____

Put 0, 1, or 2 for each item.

0 Incorrect or no response

1 correct response

2 correct and complete response

Strategy Steps	Possible Points	Points Earned
1. Comprehend problem What are you going to figure out?	2	
2. Choose strategy How will you do it—how will you solve the problem?	2	
3. Identify necessary information What information will you use to solve it?	2	
4. Solve problem correctly Solve it here.	2	
5. Explain a rule, pattern, or idea you used. What idea, rule, or pattern does your solution show?	4	
Total Points		

Plus:

You can earn more points!

Make up a parallel word problem—a problem like this one but with different information.	5	
Make a strategy guide—step-to-step explanation that another student can use to solve this kind of problem.	10	

Your Revised Total—

Add the PLUS points and the Strategy Points.

Math Problem Solver Plan

CCSS Math Practice Standard 1. Make sense of problems and persevere in solving them.

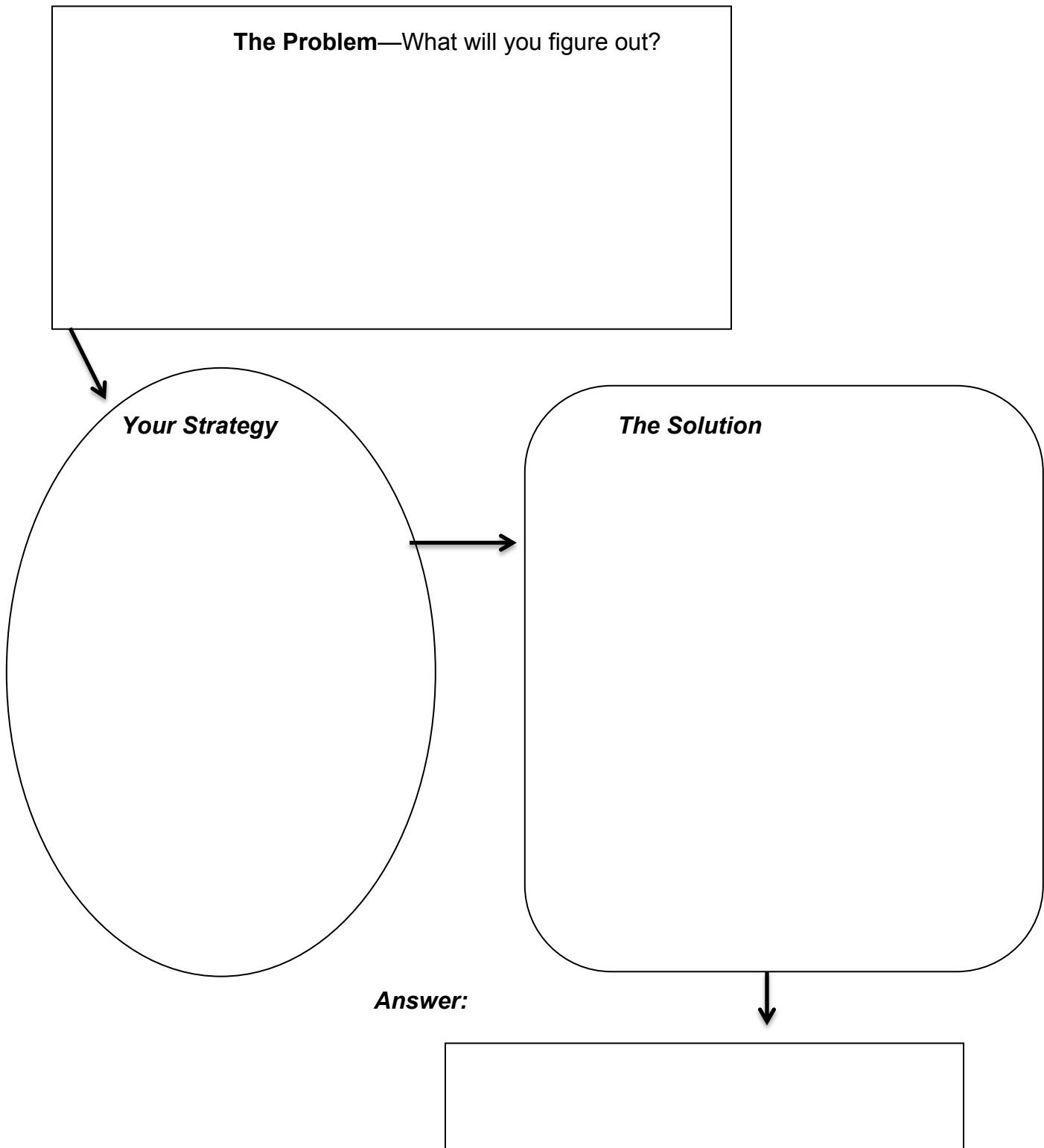
1. Read a Word Problem.
2. Complete this chart.
3. Then solve the problem.

What is the question asking me to figure out?	
How will I solve it?	
What information will I use?	

Solve the problem.
Then pair and compare.

Math Problem Flow Chart

Common Core Math Practice Standard 1: Make sense of problems and solve them persistently.



Math Problem Analyzer

Common Core Math Practice Standard 1: Make sense of problems and solve them persistently.

1. What are you going to figure out?	
2. How will you solve the problem?	
3. What information will you use?	

4. Solve it here. If you need more space use the back of the page.

5. What idea, pattern, or rule does your solution show?	
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This guide was developed through funding from the Institute for Education Sciences, US Department of Education

RESOURCES

Reinforce Math Content and Skills

Vocabulary and Learning Report	p. 36
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SMART CHART: Vocabulary and Learning Report

Common Core Reading Anchor Standard 4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings....

Anchor Standard 4 Competence target: Can define and use academic vocabulary to explain a topic.

TOPIC: _____

List five important terms you need to understand about this topic.

They should be core words—words that are important to explaining the topic.

Write a synonym or example for each word.

CORE WORD	SYNONYM or EXAMPLE

Make It Clear: Use all five core words to explain the topic.

Write a sentence for each word or a paragraph that includes them.

MATH SMART PACK

Put the words, letters, or numbers you want to learn into each rectangle. Then cut them out and use them as flash-cards or to play matching games or card games.

TIME TABLE FACTS

Put facts like these on pieces of paper. Cut them out and match them.

$$2 \times 4$$

$$8$$

$$3 \times 4$$

$$12$$

$$4 \times 4$$

$$16$$

$$5 \times 4$$

$$20$$

$$6 \times 4$$

$$24$$

OPERATIONS plus \$

Put numbers into boxes. Then cut them out and make up math problems.
Add percentages and decimal signs or algebra symbols for advanced math.

+

-

×

=

÷

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How can students use math to think across the curriculum?

NGSS Scientific and Engineering Practices

Circle the practices that apply in social science and science.

Underline the practices that require math.

- 1. Asking questions (for science) and defining problems (for engineering)**
2. Developing and using models
3. Planning and carrying out investigations
- 4. Analyzing and interpreting data**
- 5. Using mathematics and computational thinking**
6. Constructing explanations (for science) and **designing solutions (for engineering)**
7. Engaging in argument from evidence
8. Obtaining, evaluating, and communicating information

Who needs tables and graphs?
Scientists and social scientists.

The following pages can guide activities or assess student competence.

Collect Data

Task: I can construct a table and analyze the data it presents.

1. Choose your categories. Write them in the space at the top of each column.
2. Collect information.
3. List it in your table.

Title of the Table

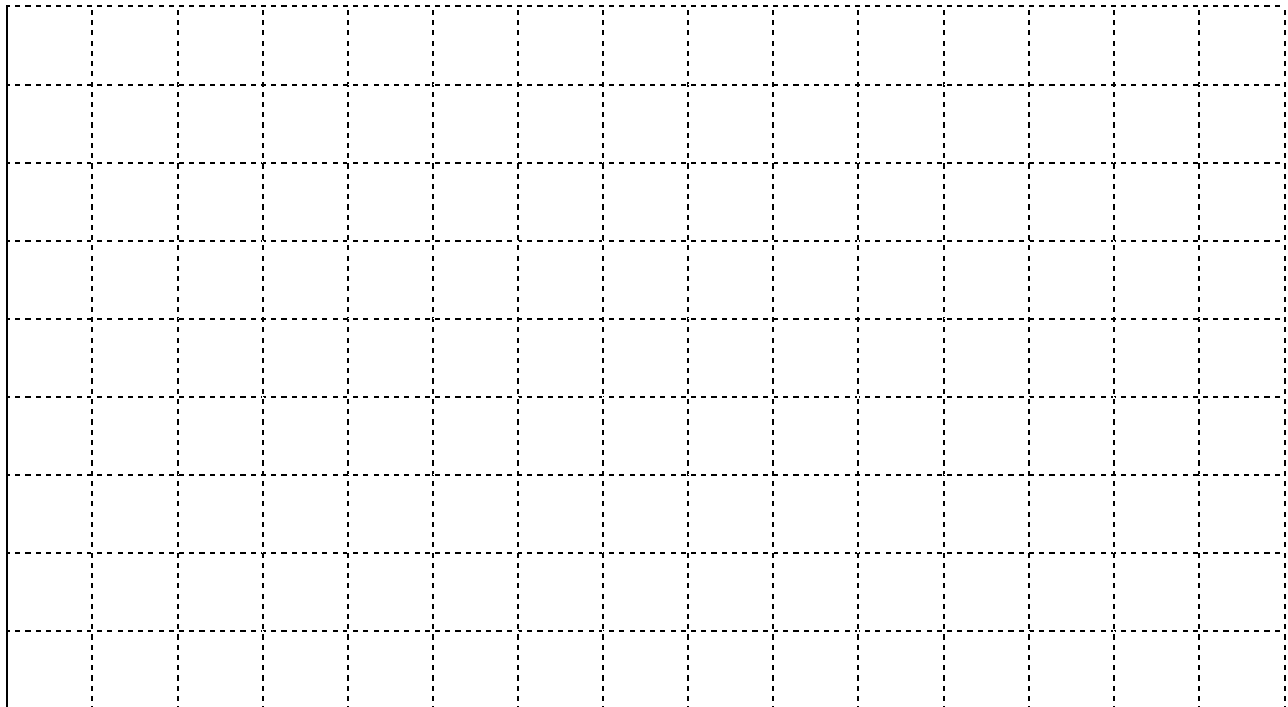
Analysis of the Table

Explain patterns or relationships you see.

I Can Analyze Relative Quantities with a Bar Graph

Locate and collect information about different quantities that are related to the same topic, such as the number of students choosing different colleges.

Title of the Graph: _____



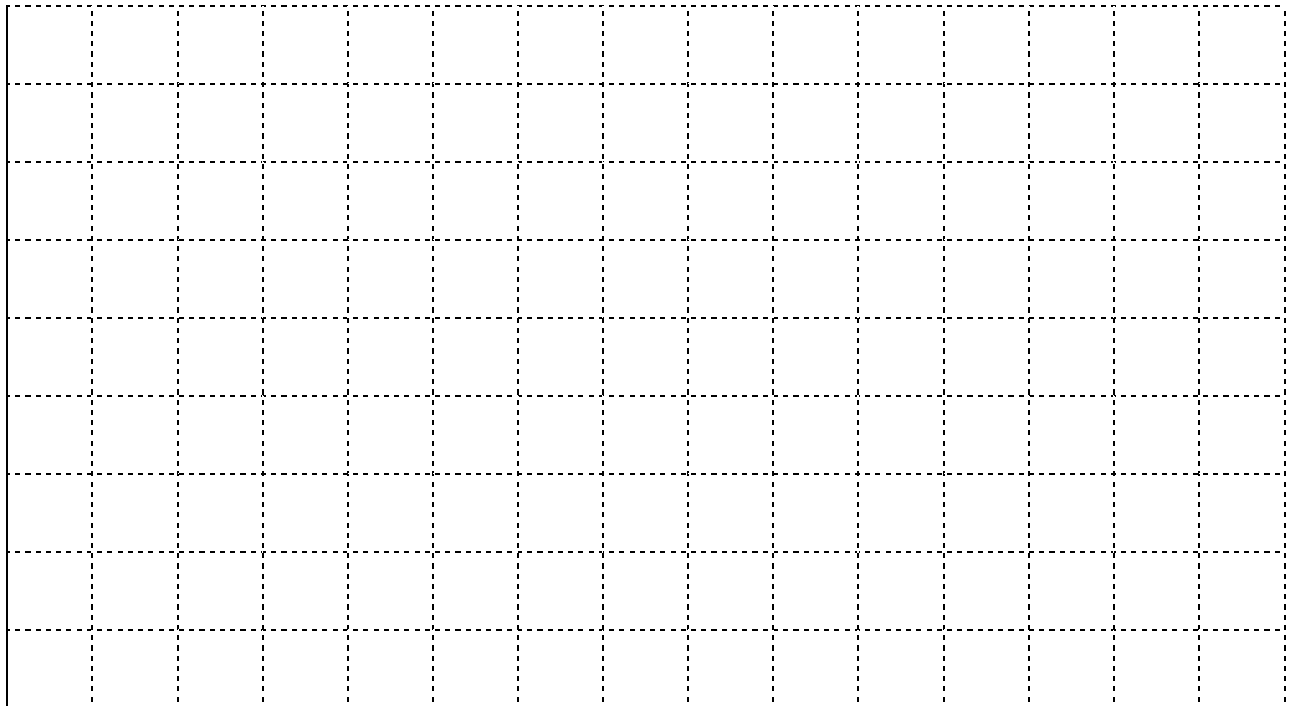
Analyze Patterns and Relationships

Explain the patterns or relationships it shows.

I Can Analyze Changes over Time with a Line Graph

*Locate and collect information about a situation that **changes over time**. Use that information to make a line graph.*

Title of the Graph: _____



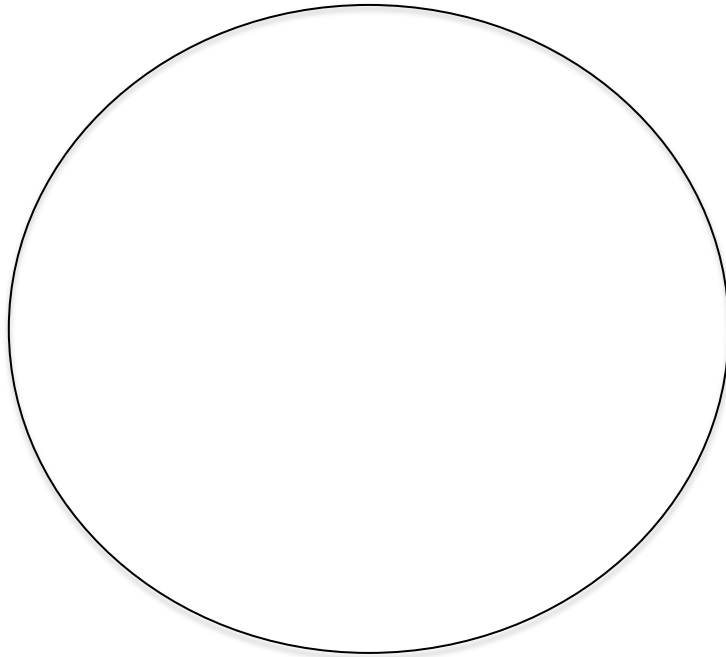
Analyze Patterns and Relationships

Explain the patterns or relationships it shows.

I Can Analyze Proportions with a Circle Graph

Locate and collect information about different quantities that are related to the same topic, such as the number of students choosing different colleges. Then figure out the proportions and make a circle graph (or pie graph) to show them.

Title of the Graph: _____



KEY:

Analyze Patterns and Relationships

Explain the patterns or relationships it shows.

Graph Challenge: Is Global Warming Affecting Chicago?

Data Source: Chicago Department of Planning and Development

1997 Average Temperature (fahrenheit)

Month	Maximum	Minimum	Mean
January	29.0°	12.9°	21.3°
February	33.5°	17.2°	25.7°
March	45.8°	28.5°	36.6°
April	58.6°	38.6°	48.5°
May	70.1°	47.7°	59.1°
June	79.6°	57.5°	68.4°
July	83.7°	62.6°	73.1°
August	81.8°	61.6°	71.9°
September	74.8°	53.9°	64.3°
October	63.3°	42.2°	52.6°
November	48.4°	31.6°	39.5°
December	34.0°	19.1°	27.1°

SOURCE: National Climatic Data Center.

Weather 1997

Average Temperature (fahrenheit)	48.5°
Warmest Day (7/26/97)	97.0°
Coldest Day (1/17/97)	-9.0°
Total Snowfall	40.1"
Total Precipitation	31.7"

Get 2015 Chicago weather data.

Make a data table that includes these data and the 2015 data.

Use a line graph to trace the changes.

Analyze the changes.

Support your conclusion

Analyze Water Usage In The City Of Chicago, And How Waste Water Is Treated.

Research by Les Nagy, Chicago History Museum, 2015

Analyzing and Interpreting Data Tables

What patterns do you see?

What causes and effects do you infer?

Table 1. The Data Below Shows Proportions Of Public Water Usage By Category At Local, State, & National Levels

Use Category	Cook County	Illinois Average	United States Average
Domestic (homes, gardens)	41 %	51 %	56 %
Commercial (restaurants, grocery)	19 %	24 %	17 %
Industrial (soft drink producers)	3 %	6 %	12 %
Public Use (parks & recreation)	37 %	18 %	15 %

Source: *Selected data taken from Chicago Metro Water Reclamation District *Monitoring & Research Department 2015*

Link: <https://www.mwrd.org/irj/portal/anonymous/WQM>

Table 2. How We Use Water: Proportions Of Home Water Usage By Category – US. Averages

Type Of Usage	Toilets	Clothes Washers	Showers	Sink Faucets	Leaks	Bath Tubs	Dish-washers	Other
Percent % Of Total Water used	26.7	21.7	16.8	15.7	13.7	1.7	1.4	2.2

Source: *Maryland Department Of Water – Via *Chicago Tribune*

Link: http://blogs.chicagotribune.com/news_columnists_ezorn/2011/11/soak.html

Table 3. Raw Sewage (Wastewater) Treated & Released At Chicago's Seven Water Treatment Facilities in 2014

Treatment Facility	Bio-Solids Recycled From Sewage *tons per year	Average Amount Of Water Treated * millions of gallons per day	Chicago Waterway Where Treated Water Is Released.
Stickney Plant	126,159	727.00	Chicago Ship Canal
Calumet Plant	24,588	255.00	Cal-Sag Channel
O'Brian Plant	34,272	232.00	North Shore Channel
James Kirie Plant	4,797	29.24	Higgins Creek
John E. Egan Plant	5,454	24.70	Salt Creek
Hanover Park Plant	816	9.00	West Branch DuPage River
Lemont Plant	304	2.39	Des Plaines River

Source: Chicago Metropolitan Reclamation District of Greater Chicago - *MWRDGC, 2014 Annual Report*

Link: <https://www.mwrd.org/irj/portal/anonymous/waterreclamation>

Link: <https://www.mwrd.org/irj/portal/anonymous?NavigationTarget=navurl://c3d6b816b04b2fce0ffd1c0f1622d8f8>

Table 4. Number Of Chicago Area Organizations And Businesses Using Recycled Sewage Bio Solids As Fertilizer In 2014

Category Of Chicago Organization	Suburban Parks	Chicago Parks	Chicago Area Schools	Chicago Area Golf Courses	Landscaping Companies
Number Of Users	2,838	3,769	284	527	743

Source: Chicago Metropolitan Reclamation District of Greater Chicago

Link: <https://www.mwrd.org/irj/portal/anonymous?NavigationTarget=navurl://c3d6b816b04b2fce0ffd1c0f1622d8f8>

Chicago Math Data: Explore Water Consumption In The United States 1950 - 2010

Research by Les Nagy, Chicago History Museum, 2015

Analyzing and Interpreting Data Tables

What patterns do you see?

What causes and effects do you infer?

Table 1. Estimated Total Water Usage Compared To Total U.S. Population (1950 – 2010)					
Year	Total Water Consumption (Billion Gallons Per Day)			*Percent Change From Previous Year	Total U.S. Population (Millions)
	Ground Water	Surface Water	Total		
1950	3	11	14	No Comparison	151.3
1955	5	12	17	+ 21 %	-
1960	7	13	20	+ 18 %	179.3
1965	8	16	24	+ 20 %	-
1970	9	18	27	+ 13 %	203.2
1975	11	19	30	+ 11 %	-
1980	13	20	33	+ 10 %	226.5
1985	14	23	37	+ 12 %	-
1990	15	24	39	+ 5 %	248.7
1995	14	26	40	+ 3 %	-
2000	16	27	43	+ 8 %	281.4
2005	14	30	44	+ 2 %	-
2010	15	27	42	- 5 %	308.7
*Use this link to learn about percent change: http://www.mathgoodies.com/lessons/percent/change.html					

Source: Water Consumption - United States Geological Survey –

Link: <http://water.usgs.gov/watuse/wups.html>

Source: Population Data - United States Census Bureau -

Link: https://www.census.gov/newsroom/cspan/1940census/CSPAN_1940slides.pdf

Chicago Area Jobs

The first column lists **some** of the **many kinds of jobs** in the Chicago area. Column 2 lists **the number of jobs in the Chicago area** in each field in 2014. Column 3 lists the hourly wage. *To get information about these jobs and many more, go <http://www.bls.gov/ooh/a-z-index.htm>*

KINDS OF JOBS	Number of Chicago Jobs	Hourly Wage
Aerospace Engineers	230	\$45.84
Anesthesiologists	1,310	\$94.26
Arbitrators, Mediators, and Conciliators	200	\$36.78
Architects	3,140	\$35.12
Audio and Video Equipment Technicians	2,090	\$19.90
Bookkeeping, Accounting, and Auditing Clerks	34,270	\$19.31
Broadcast Technicians	740	\$19.83
Camera Operators, Television, Video, and Motion Picture	660	\$27.71
Chemists	1,700	\$34.97
Child, Family, and School Social Workers	9,430	\$26.42
Choreographers	40	\$24.20
Clergy	1,180	\$20.04
Clinical, Counseling, and School Psychologists	3,040	\$31.61
Community Health Workers	1,870	\$18.31
Computer Network Support Specialists	4,290	\$30.58
Computer Programmers	16,480	\$36.41
Computer User Support Specialists	13,790	\$25.25
Customer Service Representatives	74,600	\$18.06
Dancers	420	\$19.31
Dental Hygienists	4,660	\$33.99
Dentists, General	2,550	\$62.86
Dietitians and Nutritionists	1,330	\$24.82
Editors	3,340	\$26.31
Educational, Guidance, School, and Vocational Counselors	5,930	\$31.51
Electrical and Electronics Drafters	420	\$28.44
Electrical Engineers	3,220	\$43.03
Emergency Medical Technicians and Paramedics	8,570	\$28.20
Environmental Engineers	820	\$38.70
Film and Video Editors	570	\$26.89
Food Preparation and Serving	345,110	\$10.47-21.47
Forensic Science Technicians	320	\$38.85
Health Educators	1,190	\$25.63
Healthcare Social Workers	2,760	\$24.42
Hotel, Motel, and Resort Desk Clerks	4,080	\$11.54
Interpreters and Translators	1,420	\$16.94

Real Chicago Math!

Competence: Interpreting a data table.

Multiplying decimals.

Multiplying with percentages.

Some workers do not work a full work week. They work part time.

Choose four jobs you would like to have while you are going to college.

You can work 20 hours a week in the job to earn money to pay your tuition.

Job	Hourly Wage	Weekly Earning

Choose the one job you would want to do the most.

Good news!

You do a great job.

Your supervisor gives you a raise of 5%.

What will you earn then?

Hourly wage: _____

Weekly wage: _____


Added Challenge: Make Your Own Word Problem

Make up a word problem.

In your problem use real numbers from the table.

Multiple Choice Question Maker

Question Maker: Write a challenging question here. Make it a WORD problem.



Write the correct answer and three other possible answers here:

a.	b.
c.	d.

Problem Solver:

What's the best answer? _____

Tell how to figure that out.

Strategy Selector

Here are some strategies to solve a math problem.

They all start with read the problem carefully to figure out what it asks.

- ✓ Read each sentence carefully to make sure you comprehend it.
- ✓ Decide what the problem includes that you need to use to solve it.
- ✓ Notice any numbers written as words—be sure you include them in your problem solution.
- ✓ Look for context—kinds of numbers—money, time, size.

Then use a strategy you know to figure out the answer.
Start by estimating what the size of the answer will be.

Here are some strategies you can use.

- List information you need to use.
- Use a model.
- Use a rule you know.
- Make a table.
- Make a diagram.
- See if it will take just one step to solve it or more steps.
- Choose an operation to start.
- Guess, check, then correct if I need to.
- Look for a pattern.
- Draw a picture so I see what the problem includes.
- Figure out what information I need.
- Underline the information I need and cross out the things I don't need.
- Make a graph.
- Make a list of operations—the steps to take.
- Make a table or chart.
- Work “backwards”.
- Think of two different ways to solve it, then choose the one you think will work better.
- Write it as a number sentence.

Most important: Think carefully and clearly.

Work with precision—make sure you know what the words in the problem mean.

Make sure you are careful to check your work.

That is Math Practice Standard 6. Attend to precision.

Online Math Resources for Students

Useful Online Resources Available at No Cost

Math Practice Resources and Games

To achieve success with Common Core standard 1—make sense of problems, then solve them persistently, students need math skills.

The key to getting skills is practice.

The key to keeping skills is using them to solve problems.

Here are free useful math sites you can use to help students practice math skills and then solve realistic problems.

Early Childhood Games

<http://www.ictgames.com/resources.html>

Infants and Toddlers

<https://www.pbs.org/parents/earlymath/>

Primary Games from Public Television

<http://pbskids.org/games/math/>

Math Games for Grades 3-5

<http://pbskids.org/cyberchase/math-games/>

Advanced Challenges from the Museum of Math

<http://momath.org/activities/>

Activities for After-School Programs from the Exploratorium

<http://www.exploratorium.edu/education/publications/math-explorer>

Lots of activities that will help your child practice math skills.

<http://resources.woodlands-junior.kent.sch.uk/maths/>

Great games to practice math facts.

<http://www.maths-games.org>

Math Hunt

With the help of the character “Number Cruncher” scour the web to find social studies and science information to help crack math problems.

<http://teacher.scholastic.com/mathhunt/>

Math Maven’s Mysteries

Use math to help crack open mystery cases

<http://teacher.scholastic.com/maven/>

Operations Activity Sites

Primary Krypto

Use five number cards and arithmetic operations to create the "target" number in this fun math game.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=173>

Product Game

Exercise your skill with factors and multiples.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=29>

Times Table

Practice multiplying single-digit numbers with this interactive multiplication table.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=155>

Around the World in 80 Seconds

Help Maggie fly around the world. Pick addition, subtraction, multiplication, division or a mix of all four. Each question answered correctly gets Maggie to her next stop.

<http://teacher.scholastic.com/activities/adventure/math2.htm>

Determine the Missing Operation Study Jams

Sometimes to solve a word problem or story problem, you need to figure out which symbol to use. Follow this step-by-step strategy

http://teacher.scholastic.com/activities/studyjams/math_operations/

Operations with Signed Numbers

Use these "flash cards" to work with an endless list of signed-number problems-- addition, subtraction, multiplication, and division of both positive and negative numbers.

http://www.free-ed.net/sweethaven/Math/arithmetic/SignedValues01_EE.asp

Operations with Whole Numbers

A complete set of activities featuring arithmetic operations with whole numbers.

Activities can be assigned according to level of difficulty as well as special features such as division with/without remainders.

http://207.5.42.159/sweethaven/math/pre-algebra/prealg01/ee/wholenuml02_ee.asp

Order of Operations with Integers

This is a set of endless drills that build confidence with solving integer operations that involve combinations of addition, subtraction, multiplication, and division. There are four levels of difficulty.

http://www.waybuilder.net/sweethaven/Math/pre-algebra/PreAlg01/ee/IntegerPrec01_EE.asp

Periods of Place Values

Rewrite numbers separating each period of place values with a comma

<http://www.waybuilder.net/sweethaven/Math/pre-algebra/drills/wholeNums01.asp>

Geometry Activity Sites

Proof Without Words: Pythagorean Theorem

Watch a dynamic, geometric "proof without words" of the Pythagorean Theorem.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=30>

Scale Factor

A common misconception is that when the dimensions of an object are doubled, the area is doubled, too. But this is not true! Use this applet to investigate how changes in the scale factor influence the ratio of perimeters and the ratio of areas between figures.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=176>

Shape Cutter

Draw and cut shapes, then use slides, turns, and flips to move the cut pieces around.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=72>

Shape Sorter

Sort shapes according to their properties using Venn diagrams.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=34>

Shape Tool

Draw, color, paste, slice, rotate, reflect, expand, and contract various shapes.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=35>

Tessellation Creator

Create patterns to cover the screen using regular polygons.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=202>

Turtle Pond

Estimate length and angle measure while guiding a turtle to a pond.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=83>

Interactive Geometry Dictionary: Areas in Geometry

Understand and investigate the area of the rectangle, parallelogram and triangle.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=21>

Patch Tool

Design a pattern using geometric shapes.
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=27>

Perimeter, Area, and Volume of Common Geometric Figures

http://www.waybuilder.net/sweethaven/Math/pre-algebra/PreAlg01/ee/geometry/PreGeom01_EE.asp

Geometry Activity Sites

Angle Sums

Explore the sum of the interior angle measures for various polygons.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=9>

Area Tool

Investigate how changes in the base and height of trapezoids, parallelograms, and triangles affect their area.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=108>

Circle Tool

Compare the circumference and area of a circle to its radius and diameter.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=116>

Cube Nets

Examine various two-dimensional figures to determine which ones can be folded into a cube.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=84>

Computing Pi

Compare two methods for computing pi.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=161>

Cubes

Determine the volume of a box by filling it with cubes, rows of cubes, or layers of cubes.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=6>

Fractal Tool

Explore iteration and patterns in shapes and numbers with fractals.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=17>

Geometric Solids

Manipulate various geometric solids. Color the solid to investigate properties such as the number of faces, edges, and vertices.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=70>

Time, Speed, and Distance Activity Sites

Distance to Horizon

Investigate the relationship between your height above the Earth and the distance you can see to the horizon.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=150>

Sound Sketch Tool

Sketch and quantify sound using two different representations.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=36>

Vector Investigation: Boat to the Island

Adjust the magnitude and direction of a velocity vector to "drive" a boat.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=42>

Vector Investigation: Dual Vector, Airplane Storm Chaser

Adjust the magnitude and direction of a velocity vector and a wind vector to "fly" a plane.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=43>

Word Problems: Time, Speed, and Distance

Practice your knowledge of time, speed, and distance with word problems

<http://www.waybuilder.net/sweethaven/Math/pre-algebra/dstProblems01.asp>

Probability Activity Sites

Adjustable Spinner

Create a spinner and examine the experimental and theoretical outcomes for a specified number of spins.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=79>

Fire

Simulate the spread of a wildfire using a probability applet.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=143>

Random Drawing Tool—Individual Trials

Explore the relationship between theoretical and experimental probabilities.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=67>

Fractions, Decimals, and Percentages Activity Sites

Concentration

Play a matching game with different representations of equivalent items — match quantities to their numerals, shapes to their names, or fractions to decimals and percents.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=73>

Equivalent Fractions

Create equivalent fractions by shading squares and circles.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=80>

Fraction Game

Explore relationships among fractions while playing this interactive game.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=18>

Fraction Model

Explore different representations for fractions.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=45>

Fractions, Fractions, Fractions

This is version 2.0 of the popular fractions "flash cards." Fractions are learned by looking at a lot of examples and they are mastered by doing a lot of problems.

<http://edgeroamer.com/sweethaven/math/ee/fracs03.asp>

Fractions Study Jams

Watch a video and take a quiz about understanding fractions

<http://teacher.scholastic.com/activities/studyjams/fractions/>

Free Ride

Use this applet to explore fractions using the context of a bicycle and gear ratios.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=178>

Mastering Decimal Fractions

This is a complete set of endless drills with decimal arithmetic. You can select among 24 different activities that are arranged according to type of operation and level of difficulty.

http://www.waybuilder.net/sweethaven/Math/pre-algebra/Decimals01_EE.asp

Rounding Decimal Fractions

Activities for practice of rounding decimal fractions

http://www.sweethaven.com/sweethaven/Math/pre-algebra/PreAlg01/ee/DecRound01_EE.asp

Working with Percents

Practice percent problems, converting between decimals and percents, and determining parts and rates

http://www.waybuilder.net/sweethaven/Math/pre-algebra/PreAlg01/ee/Percent01_EE.asp

Algebra Activity Sites

Algebraic Transformations

Explore commutativity and associativity within a geometric situation.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=193>

Pan Balance—Expressions

Investigate the concept of equivalence by "weighing" numeric and algebraic expressions.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=108>

Pan Balance—Numbers

Find equivalent numerical expressions using a balance scale.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=26>

Pan Balance—Shapes

Explore relationships among weights of various objects by placing them on either side of a balance scale

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=33>

Roots and Powers

Some problems require the aid of a calculator, while others encourage the learner to work the problem "on paper."

http://www.free-ed.net/sweethaven/Math/pre-algebra/RootPow01_EE.asp

Solving Linear Equations in One Variable

A list of activities for solving linear equations in one variable for independent learners

http://www.waybuilder.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp

FACTORS

Factor Game

A game that exercises your factoring ability. Test your skills against a human or the computer.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=12>

Factorize

Divide numbers into two factors, and build arrays to represent each factorization.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=64>

Graphing Activity Sites

Bar Grapher

Create a customized bar graph with your own data, or display a bar graph from an included set of data.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=63>

Box Plotter

Create a customized box plot with your own data, or display a box plot of an included set of data.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=77>

Circle Grapher

Create a customized circle graph with your own data, or display a circle graph from an included set of data.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=60>

Histogram Tool

Create a customized histogram with your own data, or display a histogram from an included set of data.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=78>

Introduction to Graphing

Here is a selection of graphic activities, from plotting points to plotting linear equations.

http://www.free-ed.net/sweethaven/Math/GraphOps/GraphPlotters/graphing02_EE.asp

Isometric Drawing Tool

Create dynamic drawings on isometric dot paper. Draw figures using edges, faces, or cubes, and then shift, rotate, color, decompose, or view them in 2-D or 3-D.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=125>

Line of Best Fit

Use this applet to plot a set of data and determine a line of best fit.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=146>

Linear Regression

Investigate a regression line and determine the effects of adding points to a scatterplot.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=82>

Mean and Median

Investigate the mean, median, and box-and-whisker plot for sets of data that you create.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=160>

State Data Map

Use color-coding to represent state information, such as population, area, and gasoline usage.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=151>

Division Algorithm

Example

Procedure

$$\begin{array}{r} 4x^3 + 11x + 4 \\ 2x + 3 \end{array}$$

1st Divide first terms.

2nd Multiply times the divisor.

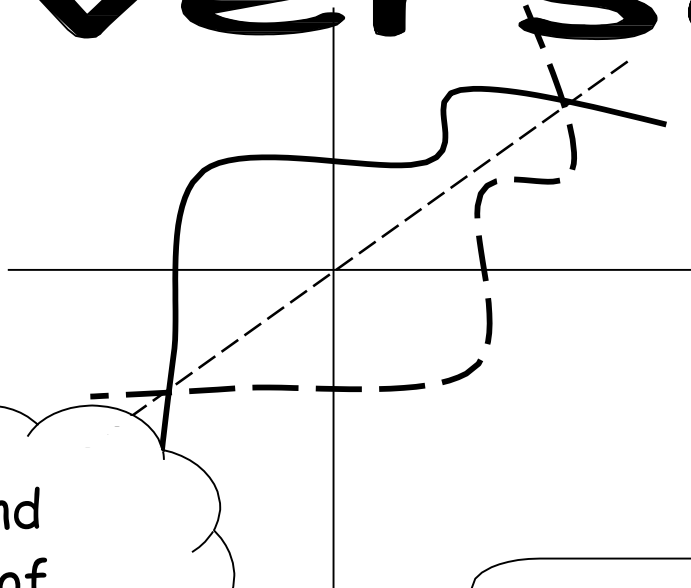
3rd Subtract by changing the signs and adding.

4th Bring down the next term and begin the process again.

Your Turn

$$\begin{array}{r} 6x^3 + x^2 + 4x + 5 \\ 3x + 2 \end{array}$$

Inverses



How do I find
the inverse of
a function?

EXAMPLE
 $f(x) = 3x^2 - 8$

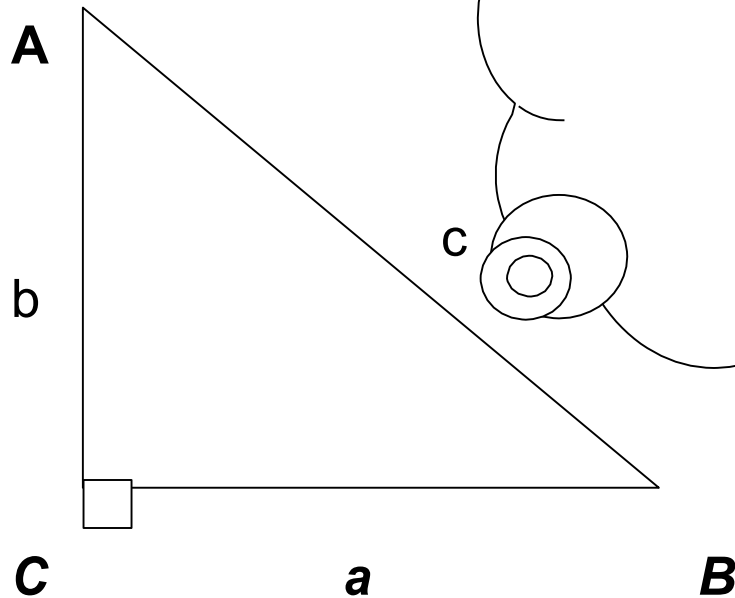
Switch to the $y =$ notation
from the $f(x) =$.

Exchange x and y in the
problem and solve for y .

Rewrite as $f^{-1}(x) =$.

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA

How do you
use the
Pythagorean
Theorem?

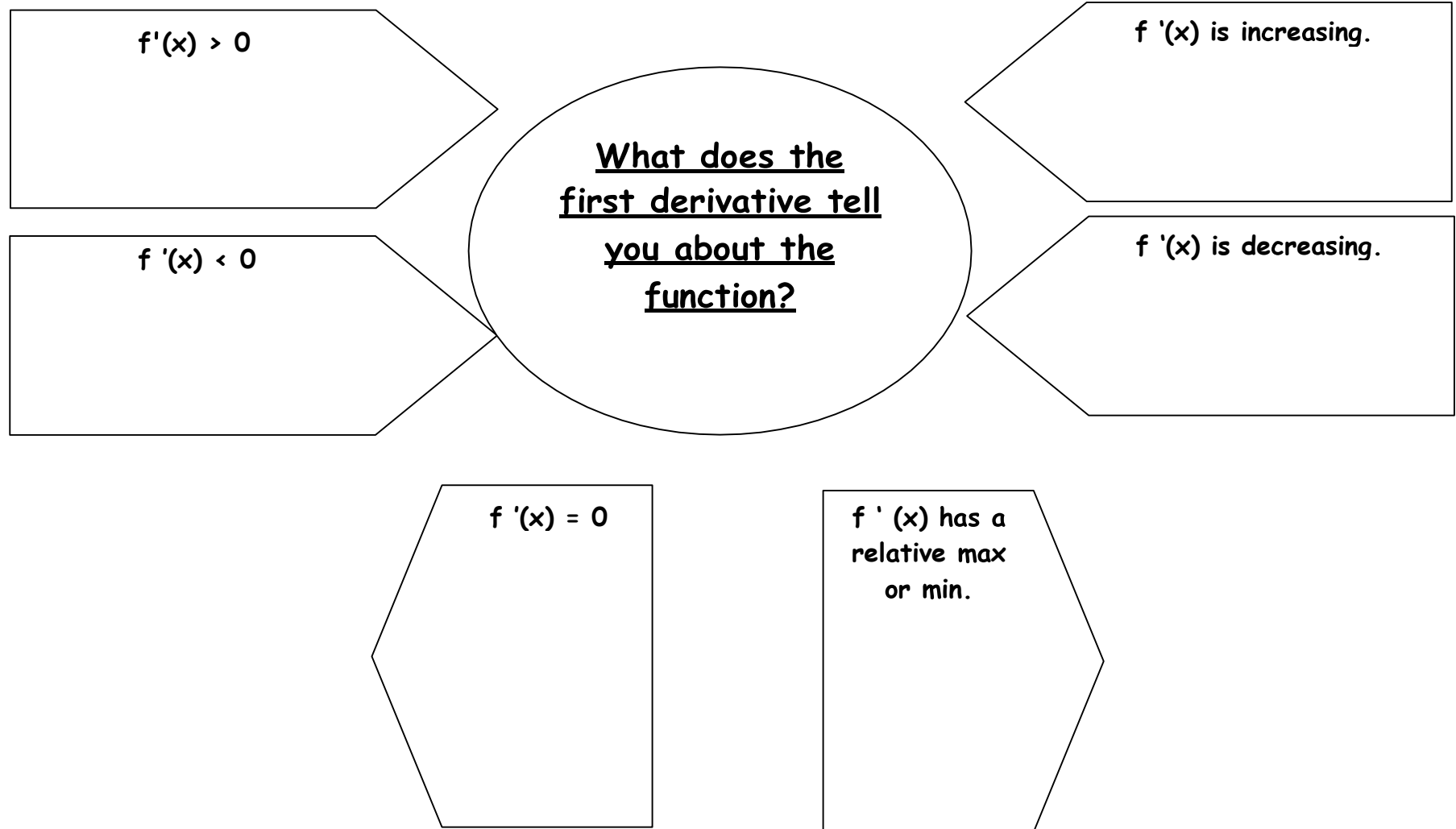


$$c^2 = a^2 + b^2$$

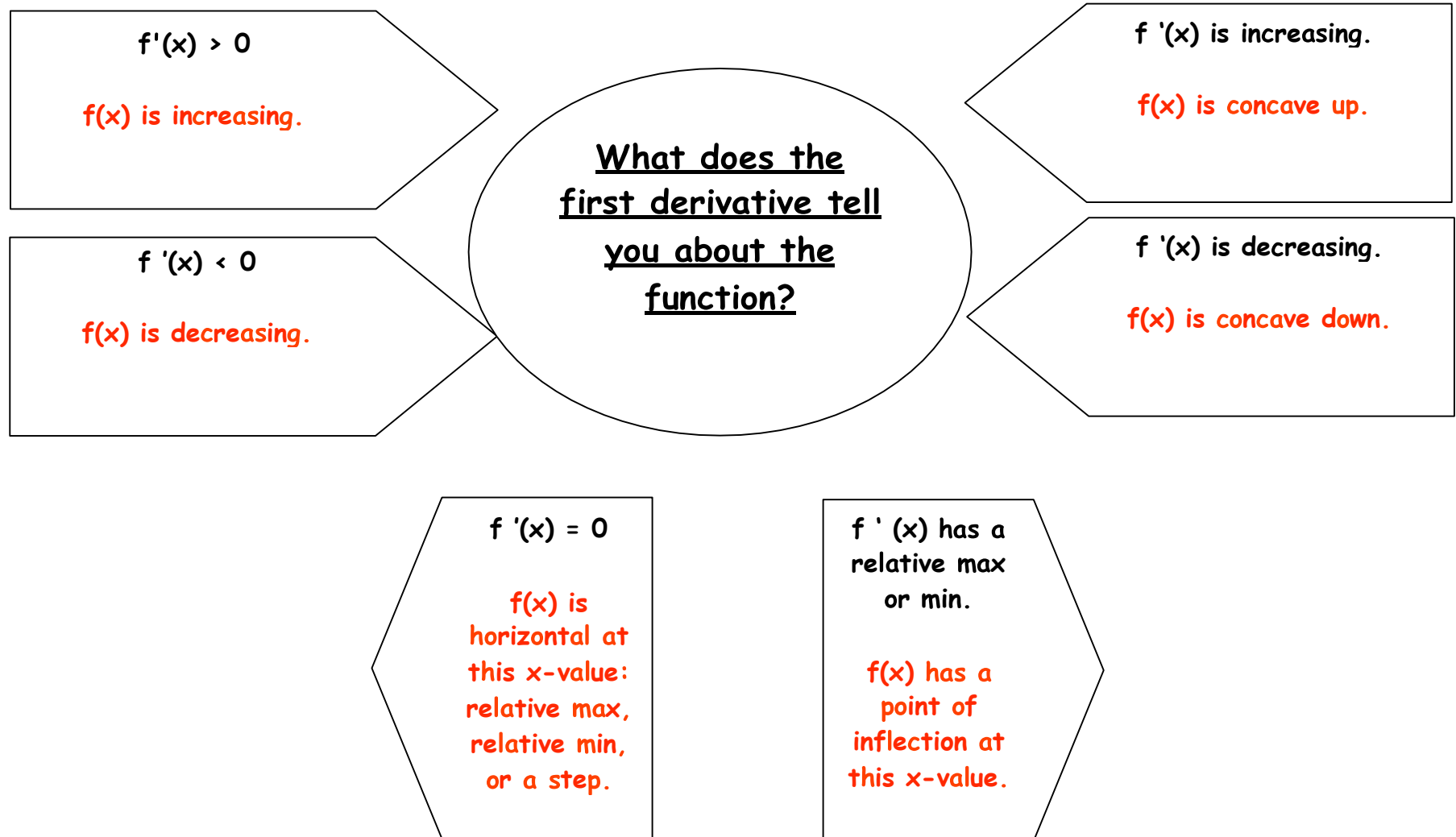
When c is unknown:

When a or b is unknown:

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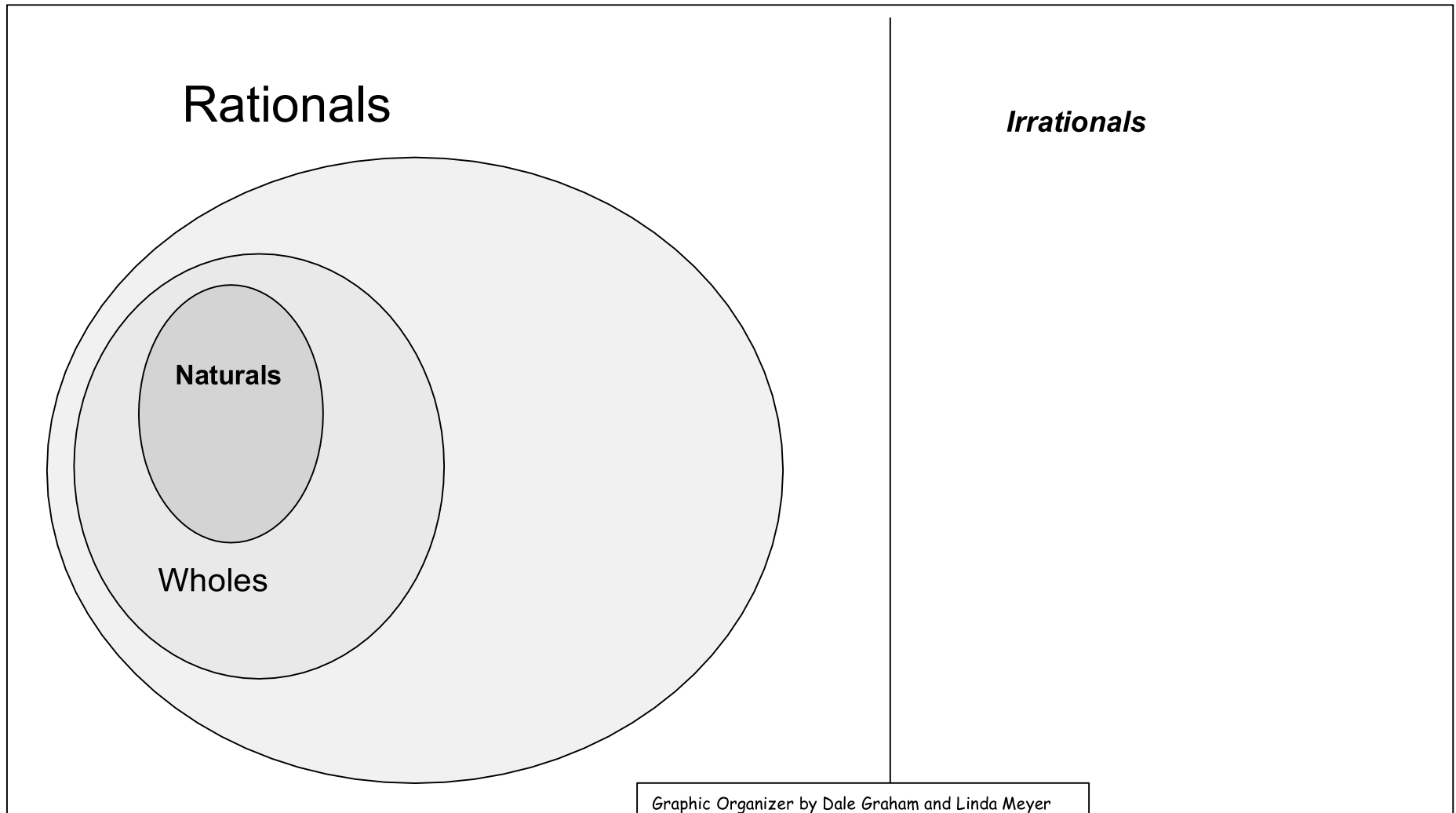


Possible Answers



What are the different types of numbers?

Real Numbers



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Thomas County Central High School; Thomasville GA

How do you find the slope of a line given two points on the line?

Given: A line that passes through (3,-6) and (1, 8)

Step 1

Step 1

Step 2

Step 2

Step 3

Step 3

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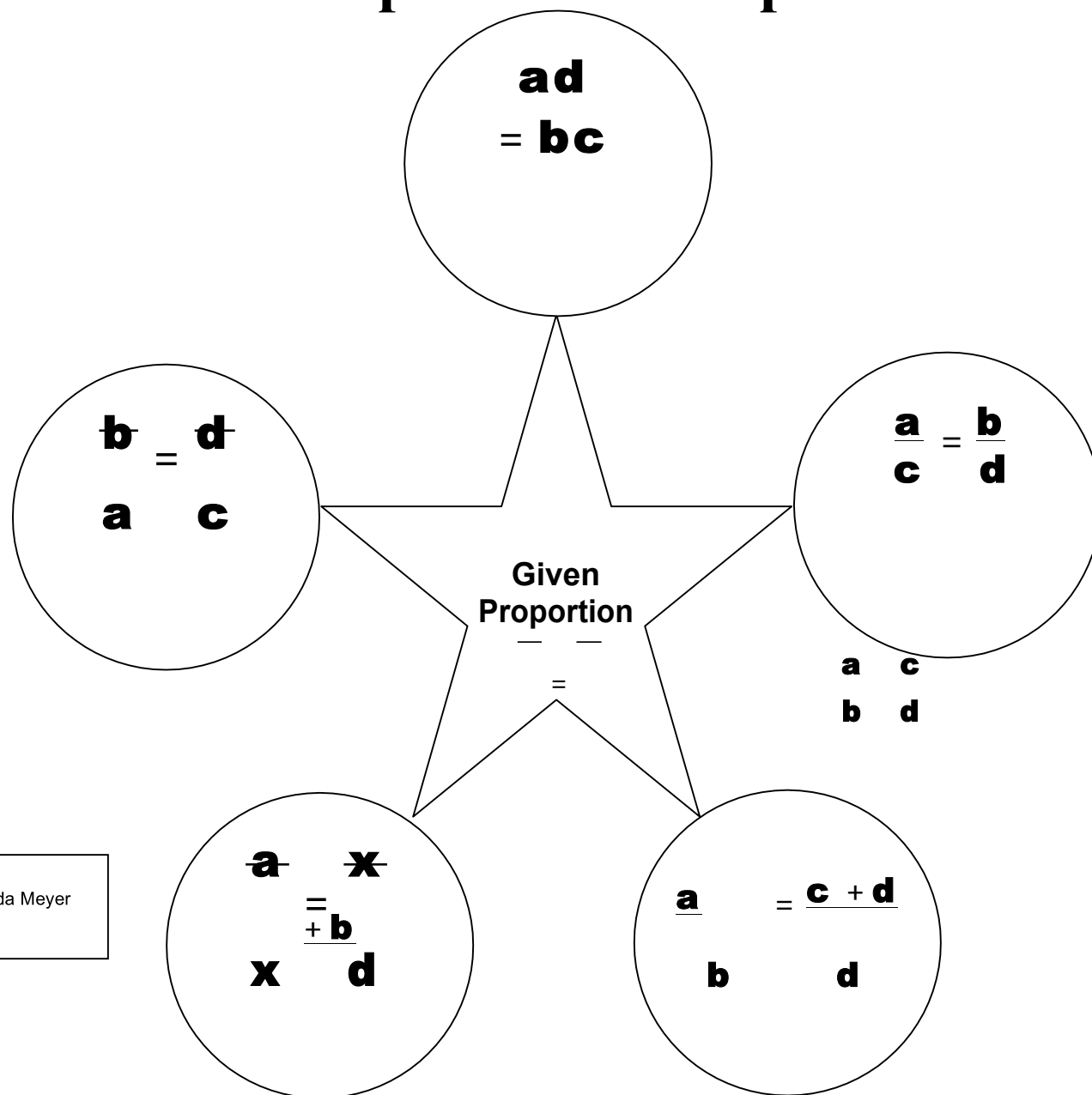
Example:

Step 1

Step 2

Step 3

What Are the Properties of Proportions?



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